Problem Set III: Due TBA

1.) Consider an ensemble of M stationary test particles in a plasma of N particles, $N >> M$ and $n\lambda_D^3 > 1$. Calculate the electrostatic energy of the field produced by the M test particles, assuming their positions are uncorrelated.

2.) Derive the growth rate of the gentle bump-on-tail instability.

a.) Develop the system as an off-shoot of the beam-plasma system. Explain when kinetic processes become relevant.

b.) Derive the wave frequency and growth rate.

3.) Derive the growth rate of the current driven ion acoustic instability.

a.) Derive the ion acoustic wave kinetically. Calculate frequency and growth. Assume no electron current.

b.) Now allow electron current, so $\langle f_e \rangle$ is a shifted Maxwellian. When is instability possible? Take ions as unshifted Maxwellian and $\delta f_e = |e| \frac{\hat{\phi}}{T_e} \langle f_e \rangle + \hat{h}_e$.

c.) Derive the marginality condition for the CDIA. What parameters control stability?

d.) Calculate the electron and ion heating.

4.) a.) Consider a chunk of collisionless, self-gravitating matter in one dimension. Here, take a "chunk" to be:

$$f = \begin{cases} f_0, & u_0 - \Delta \mathrm{v} < \mathrm{v} < u_0 + \Delta \mathrm{v} \\ 0, & \text{elsewhere} \end{cases}.$$

Here, f_0 is constant. Take $u_0, \Delta \mathrm{v}$ fixed. Using the Vlasov-Poisson system, calculate the marginal stability criterion for Jeans instability. Compare your result to the case discussed in class for a self-gravitating gas.

b.) Now consider a plasma, with

$$f = \begin{cases} f_{\max} + f_0, & u_0 - \Delta \mathrm{v} < \mathrm{v} < u_0 + \Delta \mathrm{v} \\ f_{\max}, & \text{elsewhere} \end{cases}.$$

Consider $f_0 > 0$ and $f_0 < 0$. $f_{\max}$ is the usual Maxwellian. Of course $f_{\max} + f_0 > 0$, for all v. What is the marginality condition now? Relate your result to the bunching condition discussed in class for the beam-plasma interaction. Hint: Consider the sign of the dielectric function.

c.) For collisionless, self-gravitating matter with an initially Jeans unstable distribution, discuss how the instability might saturate. Hint: Consider simple quasi-linear analysis.

5.) Consider an electron and ion plasma which is stable, but in which the electrons carry a current, i.e. assume a drift u_0. Take T_i finite.

a.) What are the collective resonances? When are they weakly damped, and approaching marginality?

b.) Estimate the thermal fluctuation spectrum ($\left\langle E^2 \right\rangle_{k,\omega} / 8\pi$) for the system described in a.).

c.) *Quantitatively* discuss the breakdown of the test particle model assumptions as the system approaches marginality as the drift u_0 increases.

6.) For the system of Problem 5:

a.) Derive the rate of electron-ion momentum transfer. What are the key dimensionless parameters determining this? Assume parameters such that the system is stable.

b.) How does increasing drift affect the transfer? Assume the system remains stable, but approaches marginality from below.

7.) Read and summarize the posted article by Rostoker and Rosenbluth on the Test Particle Model. Describe the key ideas of the Test Particle Model and how they are developed.

8.) Read and summarize the posted article by Roberts and Nielson on Saturation of the Two-Stream Instability. Explain physically how saturation occurs.

9.) Calculate the electromagnetic energy spectrum of an electron plasma at thermal equilibrium at temperature T.

10.)

i.) How far does a plasma wave packet travel at thermal equilibrium?

ii.) Describe the electric field "wake" left by a test electron in a plasma.

iii.) Calculate the slowing down time for the particle by plasma wave emission.

iv.) How does the system off-set the natural slowing down of the particle by emission? Try to calculate this and compare it to losses due to emission.